Journal
of the
British Interplanetary Society

Vol. 6, No. 6. September, 1947

THE NATURE AND DISTRIBUTION OF METEORIC MATTER

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A paper read to the British Interplanetary Society in London on 3rd May, 1947

Introduction

The purpose of this paper is to give a brief but fairly comprehensive survey of the methods of observing or detecting meteors and meteoric matter, and our knowledge of the nature and distribution of such matter obtained therefrom. Some of the topics discussed may appear, at first sight, to have little relevance to the problems of interplanetary travel, but in the light of further knowledge such irrelevances may well disappear. For example, the physical nature of meteor phenomena, discussed at some length, combined with the study of meteor light-curves, may yield valuable information of conditions of temperature and pressure in the upper atmosphere.

Meteors are, in general, small, swiftly moving celestial bodies which, ordinarily invisible, are rendered luminous when they enter the Earth's atmosphere and are heated by impact of the air molecules. They then appear momentarily as "shooting stars." Luminous "*trails*" sometimes persist in the path or track of bright meteors. Very bright meteors are known as *fireballs*, and, if explosive, as *bolides*. Meteors that partially survive the disintegrating effects of the Earth's atmosphere and fall to Earth are known as *meteorites*. Bodies similar to meteors which do not enter the Earth's atmosphere, and are detected by other means, are known as *meteoroids*, and any large-scale distribution of non-self-luminous matter is known as *meteoric matter*.

Methods of Observing Meteors

In the observation of meteors, three difficulties of a fundamental nature are encountered. Firstly, the meteor is only visible during its passage through the Earth's atmosphere, and thus we have no direct knowledge of meteoric matter which is not encountered by the Earth, although evidence of other distributions exists. Secondly, the duration of a meteor (in general, of the order of 1 to 1½ seconds) does not allow of detailed analysis, except by photography, which can be used only for the brightest meteors. Thirdly, and possibly of less importance, as the meteor track is an atmospheric phenomenon, occurring at a height of about 100 kilometres, it is in general visible only over a limited area of the Earth's surface, say 150 kilometres radius. The methods of observation reflect these difficulties, as is shown by the fact that visual observation is still, in this age of large telescopes and complex astronomical equipment, the

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principal source of information on meteors, and it is carried out entirely by amateurs. The method of visual observation is, in essence, to plot the path of the meteor in the sky with reference to the star field background, noting such characteristics as colour, brightness and time of occurrence. (The work in this country is carried out by the Meteor Section of the British Astronomical Association, and the Director, J. P. M. Prentice, has published details of the technique used by British observers.[1]) The data of heights and velocity are obtained from duplicate observations of the same meteor from two (or possibly more) stations. Telescopic observation is difficult, as clearly no previous knowledge of where a meteor will appear can be obtained, but it has its uses, as much fainter meteors can be seen telescopically than visually, and wide-field telescope systems have been used for counting faint or *telescopic* meteors. With a photographic plate, wide-angle telescopes may be used to obtain photographs of meteors, from which curves of variation of brightness of the meteor, and deviations from linearity of the path, neither, in general, visually observable, may be obtained. Due to the, perforce, short exposure determined by the meteor, only the brighter meteors can be recorded photographically. Meteor spectra are obtained with an "objective prism"—a low-dispersion prism placed in front of the object glass of a photographic telescope system. During the past few years the powerful methods of radio reflection have been applied to meteor observation, where specially designed apparatus is used to observe the reflection of radio-wave pulses (of about 5 metres wavelength) from the ionisation caused by the meteor, observation being visual or photographic on a cathode-ray tube with linear time-base. This method has the advantage that the height (or strictly speaking the range) of the meteor is determined directly on the linear scale of the tube. From the faint "travelling" echoes due to reflection off the cap of ionisation surrounding the meteor itself, the velocity may be determined directly. Further, indirect evidence of the size of meteors has been obtained by examining the strength of the meteor echoes. The method has the disadvantages that (i) the meteor's position is not known accurately, the best directive aerial system that uses a frequency that gives reflections, while at the same time being sufficiently mobile, has an angular spread of the order 15°, and (ii) only those meteors that enter the radio beam nearly perpendicular to it give strong reflections. Thus the radar apparatus deals with a different sample of meteors from visual workers at the same location, and the results of the two are not immediately comparable. Meteor radiants and orbits calculated still rely upon visual work, although "statistical" radiants have been found to a few degrees by using radar aerial systems with different orientations.[2]

Determination of Heights, Velocity and Orbit of a Single Meteor

The determination of the heights, velocity and orbit of a meteor is a matter of simple geometry, although the mathematics involved may at times be tedious. The equations used are derived in full in Olivier[3]; space does not permit, nor would any useful purpose be served, by repeating them here. The briefest outline of the principles is given, and the reader is referred to Olivier for details (or to Wylie for a survey).[3a]

The heights of the beginning and end of the path are found from simultaneous observations of the meteor's path at two stations separated by as large a distance as is convenient; for checking purposes, multiple observations are desirable. The velocity is deduced from the distance between the beginning and end points of the path, and the duration of the meteor (given by skilled visual workers to nearest $\frac{1}{5}$ sec.), assuming that the meteor's path was linear. The velocity so derived is, of course, not susceptible to much accuracy.

Before an orbit is worked out, two corrections must be made to the apparent path (apart from the obvious elimination of the orbital velocity of the Earth). The first is a correction for zenith attraction, or the effect of the attraction of the Earth on the meteor's path, so called because it is such as to make the meteor appear to come from nearer the zenith. The second correction is that due to the rotation of the Earth, known, by analogy to the similar effect with the fixed stars, as diurnal aberration. The elimination of these errors is fully discussed by Olivier. After these corrections have been applied, the velocity of the meteor at 1 astronomical unit* from the Sun is found. The path of the meteor is hyperbolic if v is greater than c, a parabola if $v = c$, and an ellipse if v is less than c, where v = velocity of the meteor at 1 astro. unit from the Sun, and c = velocity of free fall from infinity to 1 astro. unit = $\sqrt{2}$ mean motion of the Earth = (approx.) 26 miles/sec. As meteor velocities are frequently of the order of 26 miles/sec., it is sometimes difficult, owing to possible errors of observation, to say exactly whether a meteor path is elliptical or parabolic (or even hyperbolic). Thus the error of the deduced orbit may be large, but with shower meteors, where a large number of meteors travel in similar orbits (and it is only such orbits that are important) the mean orbit of the group may be determined with fair accuracy by averaging the individually determined orbits. The actual orbit is deduced analytically once the velocity and direction of motion are known. The derivation of the velocity assumes that the meteor's path is a straight line. Actually, due to zenith attraction it is a hyperbola, and due to variations in atmospheric conditions at different heights, discontinuities in direction of motion occur. These effects are very rarely noticed visually, but may frequently be seen on photographs. The retarding effect of the atmosphere is not visually observed for the average meteor, but some retardation of fireballs is occasionally seen.

Physical Theories of Meteor Phenomena

It is appropriate to consider the nature of the effects occurring in the upper atmosphere due to the passage of the meteor. A number of theories, akin to each other, have been proposed; none can be said to be completely satisfactory. Schiaparelli was the first to suggest that the impact of the air molecules caused the meteor to heat up and become luminous. This was elaborated by Öpik, who supposed that of the total energy of the meteor, only a fraction $(1 - r)$ of the energy was available for heating, the other r being dissipated into the resisting medium. The first detailed theory published was that of Lindemann and Dobson.[4] They supposed that when the meteor enters the low-density

*1 *astronomical unit of distance = mean distance Earth–Sun* = 149,500,000 *kms.* = 92,900,000 *miles.*

upper atmosphere, the energy of the air molecules impinging directly upon the meteor's surface was not available for heating the meteor. This direct impact will occur while the density of the medium is low enough for the mean free path of the molecules, λ to be of the order rv/v_0, r being radius of the meteor, v its velocity, v_0 the mean velocity of the air molecules. Lindemann and Dobson deduce that, for pure N_2, direct impacts occur for densities $\rho \leqslant 5{\cdot}6 \times 10^{-4})/rv = \rho_c$. If $\rho > \rho_c$, the meteor forms a cap of air in front of itself by compression, which the authors assume to be adiabatic. From this assumption, and the standard gas laws, they derive a formula connecting the temperature of the meteor's surface, T_2, the original temperature of the air T_0, temperature of air after compression, T_1, and the heat which flows to the meteor per second, $M = \frac{1}{2}mv^2(V_1 - V_2)/3v$, where V_1 = mean velocity of the air molecules at $T_1 = \sqrt{3RT_1/M_0}$, $V_2 = \sqrt{3RT_2/M_0}$, i.e. $M = kE$, where E is total initial kinetic energy of the meteor. The rest of this energy is dissipated as radiation. k is calculable, and found to be of order 10^{-2}. Thus nearly all the energy is expended as radiation, which conclusion is important in calculating the masses of meteors (see later). Formulae are derived to obtain atmospheric densities at great heights from the characteristics of meteors occurring at these heights, the only assumptions made being that the meteors have similar composition to the meteorites found on the Earth.

Lindemann and Dobson's theory gives much higher temperatures for the upper atmosphere than those usually accepted (see, e.g., Humphrey[5]), and Sparrow has given a theory[6] in which *NO* air cap is formed. He believes, in contradiction to Lindemann and Dobson, that direct molecular impact can heat up the meteor at heights of 70 to 100 kilometres, so that the meteor's surface is heated to a temperature T, radiates according to Stéfan's Law, and commences to evaporate. As evaporation becomes copious, the accession of energy is much increased, and the meteor becomes visible. The theory gives results in much better agreement with accepted atmospheric data. Sparrow also attributes Lindemann and Dobson's anomalous results to the, in his opinion, unjustifiable assumption of adiabatic compression.

The third theory of importance is that of Maris.[7] He concludes that the kinetic energy of the meteor is transformed into radiation, not at the meteor's surface (which he finds would have, as a black body, to be at 19,000° K., the material actually vaporising about 2,000° K.), but in the surrounding medium. The mechanism postulated is that the impact between a molecule of atmospheric nitrogen, say, and the meteor moving with velocity of 4×10^6 cm./sec., would have enough energy to eject 50 molecules of iron from the solid body of the meteor. As the energy of the average meteor is dissipated in about 1½ seconds, Maris concludes that the energy of the meteor is changed to radiant energy by high-energy molecules ejected from the meteor, which communicate their energy to the air molecules. The meteor flashes into view when the energy of the escaping molecules is sufficient to support the energy of the train. Impacts between air molecules in the path of the meteor and escaping molecules prevent an increase in brilliancy as the meteor moves from the height of appearance to denser strata of air. At a height of 60–80 kms., an air cap

forms, which then provides the luminosity of the meteor, but reduces its velocity. On this theory, less than 0·02 of the energy of the meteor will be expended directly against air resistance, more than 0·98 being spent in ionisation and excitation beyond the main mass of the meteor. Maris' theory gives a possibility of meteors occurring as high as 500 kms. Here, in the rare atmosphere, the explanation of radiation from the meteor's surface and/or an air cap is not tenable, while the "explosion" mechanism of Maris is still possible. In this connection, it is worth noting that some observers have believed that some telescopic meteors have occurred at such heights[8] and also, a very few echoes, apparently similar to the normal echoes from meteors, are seen on radar equipment to occur at heights of about 400 kms. (Their identification as an effect of meteors is still in doubt, however.) A second point of great importance in connection with Maris' theory concerns the nature of the emitted radiation. Visible radiation is emitted by energies of 2–4 electron-volts, while the energy of complete inelastic impact of an air molecule with the meteoric mass is generally of the order of 150 electron-volts, equivalent to the ionising potential of soft X-rays; we would thus expect a large proportion of the radiation to be located in the ultra-violet or soft X-ray region of the electromagnetic spectrum. Some of this energy would be absorbed by the surrounding medium, and re-emitted as visible light, but this would only be a few per cent. of the total.

The discussion above gives a sufficient idea of the measure of agreement, and at the same time the measure of disagreement, between rival theories of the nature of the effects of a meteor on the atmosphere. It bears relevance to the calculation of meteor masses, given later. A general survey of the physical problems involved has been given by Öpik in 1937.[9]

Physical Observation of Meteors

The spectra of meteors have been examined in spite of the comparative difficulty of obtaining sufficient dispersion combined with sufficient intensity. Of course, only the brighter meteors can be so examined. In general, the meteor itself shows a line emission spectrum superposed upon a continuous thermal spectrum. In some spectra the continuous spectrum is absent, probably owing to lack of sensitivity of the photoplate used. Such observations agree qualitatively with the type of theory given above, but the brightness of the emission spectrum is, if anything, in favour of Maris' theory, in which ionisation (and hence subsequent line emission) plays the major part. The spectrum of a meteor trail (the faint glow which remains in the path of some meteors after the meteor itself has passed, and which may remain any length of time from a fraction of a second up to half an hour or more in very exceptional cases) is difficult to record owing to its faintness, and also to the fact that, on a photoplate, the light of the meteor that has passed swamps the light of the trail. Such observations that have been taken indicate that the trails exhibit line spectra. The continuous spectrum *may* be absent due to faintness, but only an electrostatic origin appears to be capable of explaining the long duration of the trail, and the hypothesis that the luminosity of the trail is due to recombination of the atoms ionised by the passage of the meteor, or else left behind by the meteor, seems satisfactory. The duration of the radar echoes

also indicate that much ionisation remains in the path of the meteor after its passage, long duration echoes of constant range being observed. (A new detailed theory of ionisation in the meteor trail is being worked out by Herlofsen, but so far nothing has been published.)

Chemically, the meteor spectra indicate two types of meteor. Spectra of Type Y (after Millman[10]) show the lines M and K of ionised calcium, the iron lines being present but not pronounced; Type Z, the less frequent, show predominantly iron lines, calcium apparently absent. The distinction is not well defined, and some spectra occur which it is difficult to classify. That these results agree closely with the dual nature of meteorites (Stony meteorites and Iron meteorites) is a sure clue to the origin of the latter, which are, of course, exceptionally large meteors that have partially survived passage through the atmosphere. Traces of magnesium, aluminium, manganese and chromium, detected by chemical analysis in meteorites, have been observed in meteor spectra. No correlation between spectrum and observed colour of the meteor has been identified (showing the importance visually of the continuous spectrum), although magnesium meteors appear to have visually a greenish tint, probably due to the pronounced green triplet, $\lambda\lambda 5167, 5173, 5184$. No correlation between the spectra of meteors in a single shower has been observed, either.

The theories referred to above may be used to investigate the upper atmosphere by studying meteor light-curves, which must be examined photographically, with the help of a recording microphotometer. So far, the investigation has been mainly in classifying the various curves into different types (e.g. Höffleit[11]). In general, the light-curves may show a more or less steady increase to a maximum in the middle of the path, and then gradually fade again, or else the luminosity stays approximately constant over most of the path, with a sudden burst near the end of the path. (In this connection, it is worth noting that Maris' theory predicts that such a burst might occur with bright meteors when the air cap forms.) More information can be obtained if the deceleration of the meteor can be observed. We have seen that the integrated light of the meteor is a measure of its mass. The instantaneous brightness of the meteor may be taken as proportional to the rate of loss of matter by the meteor. It is thus possible to determine the mass of the meteor at any point on its photographed track, and if the deceleration of the meteor at this point can also be observed, then the density of the atmosphere can be determined since the resistance to a body of known mass is calculable. In order to observe this deceleration, experiments have been performed[11a] with two cameras at different locations, equipped with rotating shutters operated by synchronous motors, these shutters breaking the photographed track into segments equally spaced in time, so that the angular velocity of the meteor at any point on the track can be measured directly.

Before leaving the physical effects of meteors, mention must be made of their effect on short-wave radio systems, for which subject there is now a large amount of literature; for example, the pioneer papers of Skellett.[11b] The interference near the receiver between the direct beam and the beam reflected off the ionised cap of the moving meteor (whose frequency is slightly altered by

Doppler effect) gives rise to audio-frequency beats, the now well-known "meteor whistles." The relative deceleration of the meteor produces a change in pitch of the note, and the effect may possibly be useful for determining decelerations.

Distribution of Meteors in Space

The distribution of meteors in the space traversed by the Earth in its orbit can be deduced from observations of the frequency of meteors encountered at all times of the year. It must be realised that the distribution is so far from uniform that such a calculation can at best yield only orders of magnitude, these indeed being the only quantities that have any real meaning. Furthermore, the simple calculation can apply only to visual meteors, that is to meteors above a certain mass. For smaller masses, i.e. telescopic meteors, estimates of their relative abundance have varied so enormously (from, say, 50 times visual rate to some millions of times, depending upon the observer, the instrument used, its location, etc.) as to be almost meaningless. The possibility of a law of distribution of mass being derived, and a tentative suggestion for such a law, is discussed in the next section. Here, the calculations will be restricted to visual meteors, i.e. meteors above magnitude 3, approximately.

As data the following table, given by Hoffmeister (see Olivier[12]), has been taken. They refer to the hourly rates of visual meteors for one observer:—

Average monthly variation		Average hourly variation	
Jan.	7·3	6½ 7	4·1
Feb.	6·0	7– 8	4·5
Mar.	7·7	8– 9	5·2
Apr.	6·9	9–10	6·8
May	6·1	10–11	7·7
June	6·0	11–12	8·6
July	(12·0)	12–13	10·6
Aug.	9·5	13–14	11·3
Sept.	11·4	14–15	12·3
Oct.	14·5	15–16	12·1
Nov.	13·3	16–17	12·4
Dec.	10·8	17–18	15·4

It is immediately apparent that the number of meteors observed is markedly greater during the second half of the night, and during the second half of the year (although a less pronounced variation). These effects are due to the rotation and motion of the Earth. Before midnight at any given point on the Earth's surface, the observer is on the following side of the Earth with respect to its revolution, and is protected from all meteors except those that overtake the Earth, while after midnight the observer is on the forward side and fully exposed to the bombardment. The maximum of the annual variation occurs in the autumn (in the Northern Hemisphere), for at this season the point towards which the Earth is revolving is highest in the heavens at night. The sudden increase after midnight is easily noticeable to anyone during a prolonged watch.

The figures given represent average hourly rates for sporadic meteors.

Some nights, however, the hourly rate may be much higher than that given, and if, on such a night, the meteor paths are plotted, the direction of flight of most of the meteors will be seen to pass through a single point (or, more strictly, through a very small area) of the sky. Such a point is known as a radiant point, and the meteors associated with it as a meteor shower. No meteor path of the shower actually passes over the radiant, and in general the further the path is from the radiant, the longer it is. This radiant effect is actually one of perspective, the meteors of a shower moving in space in parallel (or nearly so) orbits. This is clearly shown in Fig. 1, where O is an observer on the Earth's surface SON, AA^1, BB^1, ... FF^1 are paths of meteors of a shower in the Earth's atmosphere, and aa^1, bb^1, ... ff^1 are the apparent paths of the meteors among the stars, i.e. the projection of their actual paths on to the celestial sphere. As is seen, all these apparent paths pass through the "vanishing point" R, which is the radiant. This is clearly true in three, as well as two, dimensions.

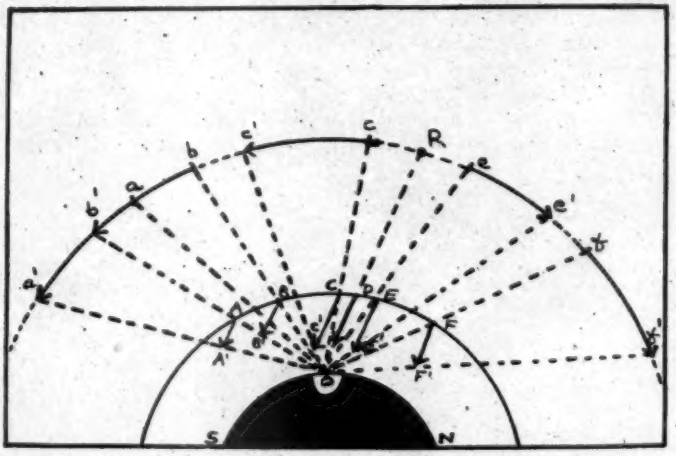

FIG. 1 Radiant Effect

The number of meteors in a shower depends entirely on the shower. Some may be so diffuse that the rate is hardly perceptibly above the normal sporadic (i.e. non-shower) rate, while the rate, in exceptional cases, may reach a thousand an hour or more visually. The nature of a meteor shower is discussed later; the distinction between shower meteors and sporadic meteors is important in connection with the space distribution of meteors.

In order to calculate the space distribution of meteors, let us assume that a meteor is visible a distance x on the Earth's surface from the point where it is at the zenith. Then, if the hourly rate for one observer is ρ, the number of visible meteors in a cylinder, radius x, length μ, may be taken, to a sufficient approximation, as ρ, where μ is the mean hourly motion of the Earth. Thus there is, assuming even distribution, one meteor in a volume $\pi x^2\mu/\rho$ which

becomes a visible meteor, i.e. a meteor brighter than magnitude 3. x may be taken as 50 miles = 80 kms. (a very low estimate); μ = 66,600 miles = 107,280 kms. The actual hourly rate will be somewhat higher than the observed rate, as an observer is unable to watch the whole sky at once, but, on the other hand, he will certainly see meteors farther than 50 miles away, and these factors are probably of little importance. Table I has been constructed in this way to give the volume occupied by 1 visual meteor in sporadic distribution, and in showers of various intensities. As mentioned above, a visual meteor may be taken as one whose stellar magnitude (in the Earth's atmosphere) is greater than three. In order to calculate the actual distribution of meteors it is necessary to know the distribution in magnitude of meteors down to the very faint meteors. The question is related to the masses of meteors, and is discussed in the next section. The above discussion was intended to show the approximate density of meteors, and the difference between the sporadic and shower densities.

TABLE I

SPACE DISTRIBUTION OF VISUAL METEORS (BRIGHTER THAN MAG. 3)

Types	Maximum visual hourly rate	Volume containing one meteor (cubic mls.)	(cubic kms.)
Sporadic	15	$3{\cdot}5 \times 10^7$	$1{\cdot}4 \times 10^8$
Minor shower	30	$1{\cdot}8 \times 10^7$	$7{\cdot}2 \times 10^7$
Major shower	100	$5{\cdot}5 \times 10^6$	$2{\cdot}2 \times 10^7$
Major shower (prolific)	500	$1{\cdot}1 \times 10^6$	$4{\cdot}3 \times 10^6$
Major shower (very prolific)	2,500	$2{\cdot}2 \times 10^6$	$8{\cdot}6 \times 10^6$

It is now appropriate to consider the nature of a major shower. As explained, a shower consists of a group of meteors travelling in nearly parallel paths. The mean orbit of such a group may be found from the determined orbits of its individual members, and it was soon realised that some meteor showers had orbits agreeing closely with orbits of known comets. Table II gives some examples of such coincidences. A physical connection between meteor showers and comets thus appears indicated, and a comet has actually been observed to give rise to a meteor shower. The classic example is that of Biela's Comet. At its return in 1846 the comet became elongated and finally divided, each part becoming a separate comet with a nucleus and a short tail. In 1852 the two comets again came into view, travelling side by side, but separated by a distance of 1½ million miles. First one and then the other became the brighter. The twin comet has never been seen since, although there has now been time for a dozen returns. The Andromedes, or Bielid meteors, moving in the path of the lost comet, which then crossed the Earth's orbit, gave brilliant displays in 1872 and 1885. It thus appears that a comet may be considered an aggregation of meteoric matter which it sheds along its orbit (possibly only in the neighbourhood of the Sun), the matter so ejected forming a meteor stream which, if the orbit intersects that of the Earth, gives rise to a display of meteors. The exact nature of the connection between comets and meteor streams is not known, and is the subject of some controversy, but

there appears every reason to expect a meteor stream, and hence a high density of meteoric matter, in the orbit of any major comet.

The distribution of meteors in the orbit of a stream is of interest. Some showers occur prominently only once every few years (e.g. the Bielids every 6 years, 1872 and 1885 being good displays), while others occur with more or less the same intensity every year. Also, some showers have very sharp maxima (e.g. the Giacobinids, about 6 hours only), while others rise up to and fall from maximum gradually (e.g. Perseid maximum lasts several days). In general, the streams which show the most condensation along the orbit are also the narrowest streams, and probably this is due to the varying ages of the

TABLE II

METEOR SWARMS AND COMETS HAVING NEARLY IDENTICAL ORBITS

Meteor swarm	Date of maximum	Associated comet	Comet's period
Lyrids	April 20	1861 I	415 years
η Aquarids ..	May 6	Halley's	76 ,,
Perseids	August 11	Tuttle, 1862	123 ,,
Draconids	October 9	1900 III	6½ ,,
Leonids	November 14	Tempel, 1866	33 ,,
Andromedes ..	November 24	Biela's	6¾ ,,

streams, the older streams having dispersed laterally and along the orbit. The most condensed (i.e. the youngest) streams are those most clearly associated with known comets. In the case of the Bielids, the maximum density in the orbit appears to occur in the position of the old comet, but with the Giacobinids, the maximum appears to be quite an appreciable distance behind the associated comet (Giacobini-Zinner). Meteor shower orbits suffer perturbations by the major plants, as do cometary orbits, and some streams have been lost to the Earth by such perturbations (e.g. the Bielids and a swarm associated with Comet Pons-Winnecke). The Leonids, although not lost, do not give the magnificent displays of "November meteors" that they used to, owing to perturbations by Jupiter having diverted the main part of the stream away from the Earth's orbit.

Having considered the nature and origin of shower meteors, a few words may be said on the origin of sporadic meteors. On almost any tenable theory of the formation of the solar system, some form of distruption is assumed, and a large amount of stray matter might be expected to be still pursuing orbits round the Sun, more or less independently, and such matter would exhibit itself as sporadic meteors with elliptical or parabolic orbits, i.e. with velocities (at 1 astro. unit) < 26 miles per sec. One might also expect such matter to have greatest condensation in the plane of the ecliptic, a possible explanation of the Zodiacal Light (see later). Some meteors, however, have hyperbolic orbits. These may be meteors which have been thrown into such an orbit (from an elliptical or parabolic orbit) by planetary perturbations. It has also been suggested that some of these may have originated from outside the solar system, being meteors of some other "solar" system which have been ejected by perburbations of some kind. Such a theory is rather doubtful, postulating

as it does a large number of other planetary systems, and, although a little more plausibility may have been added to it by the discovery of bodies of planetary mass associated with two of our nearest neighbours in space (61 Cygni and 70 Ophiuchi), a more acceptable hypothesis would appear to be that these hyperbolic sporadics are the particles of a cosmic dust cloud through which the Sun and solar system is passing. Such clouds are visible, either as dark obscuring nebulae such as the "Coal-Sack" in the Milky Way near the Southern Cross, or as bright reflection nebulae, as in the Orion nebula for example (see later). As to the relative numbers of hyperbolic meteors, and the interpretation to be put upon them, considerable discussion has been and still is being devoted to this subject. The British observers, whose results have yielded the greatest consistency of any samples, find the number of observed hyperbolics is small enough to be adequately explained by the above suggestions, coupled with some slight observational error in some cases. (It must be realised that it is no easy matter to distinguish between a parabolic orbit and a hyperbolic orbit of low eccentricity, as are all hyperbolic meteor orbits.) Other workers, notably Hoffmeister, on the other hand, from various data and from certain doubtful theoretical conclusions from the observed seasonal variations of hourly rates, have deduced distributions of hyperbolic meteors indicating interstellar streams. This conclusion must be viewed with the greatest caution, but appears at least worthy of mention. In parenthesis, it should be pointed out that on physical theories of meteor phenomena, the hyperbolic meteors, having the greatest velocities, should occur at the greatest heights. Visually, the Leonids (obviously elliptical orbits) appear highest, in general, and this in itself would preclude any large number of hyperbolic orbits.

Distribution in Mass

The calculation of the mass of a meteor is closely bound up with the physical theory assumed for the explanation of the meteor phenomena in the atmosphere, and for this reason alone a prominent discussion of such theories would be warranted. In general, we may say that a meteor of mass m travelling with a geocentric velocity v enters the Earth's atmosphere with an available kinetic energy $= \frac{1}{2}mv^2$. A certain fraction $(1 - k) = K$ is assumed transformed into radiation, so that if the total radiated energy is E, $K \cdot \frac{1}{2}mv^2 = E$, or $m = 2E/Kv^2$. If we let $K = 1$, then the mass m is as small as possible, $m = m' = 2E/Kv^2$. m' is then known as the "minimum mass" of the meteor, which is the smallest mass required to provide sufficient energy for the observed energy, all the initial energy being transformed into radiation. Now we have seen that on the theories discussed above, k is of the order 10^{-2}, and we may thus, without appreciable error, assume that the actual mass of the meteor is the same as its minimum mass. It should be noticed, however, that while on the theories of Lindemann and Dobson, and Sparrow the total energy emitted is taken to be, without large error, confined to the visible spectrum, on Maris' theory a large proportion of the energy will be expended as invisible ultra-violet or soft X-ray radiations. Maris' theory thus gives a higher value for the meteor's mass than the other theories. Somewhat arbitrarily, Maris takes the total energy of the emitted radiation to be 10 times the energy of the visual radiation, thus

obtaining masses 10 times the value of Lindemann and Dobson, or Sparrow. As standard, we may take the first magnitude meteor defined by Lindemann and Dobson, namely a meteor travelling with velocity 4×10^6 cm./sec. with duration 1·5 seconds, whose apparent brightness at its mean height $1{\cdot}5 \times 10^7$ cms. is that of a first magnitude star. The rate of energy generation, assuming, first that only visible radiation is emitted, is then $3{\cdot}3 \times 10^{10}$ ergs./sec. The total emitted energy is thus approximately 5×10^{10} ergs. and the mass of the meteor $6{\cdot}25 \times 10^{-3}$ grms. On Maris' theory, total energy emitted $= 5 \times 10^{11}$ ergs., and the deduced mass $6{\cdot}25 \times 10^{-2}$ grms.

It would thus appear possible to deduce the distribution in masses of meteors by measuring the distribution in magnitude, reduced to standard height, and statistically all velocities could be considered equal to the parabolic velocity, the individual velocities being all of this order of magnitude. Simple in theory, in practice it is by no means simple, as the only method of obtaining heights that is convenient for the large numbers of results required for statistical purposes would be the visual method with large numbers of observers, and visual observers cannot give the magnitude to a very high degree of accuracy ($\frac{1}{4}$ mag. at best). However, some work in this way has been done, and Öpik,[13] from quasi-observational data, has given the frequency of meteors brighter than magnitude $j = N_j$ in the form $N_{(j+1)}/N_j = \text{constant}$. Öpik gave the value of this constant as 4; other workers have suggested 3 and 5 as equally good. Öpik applied this rule to meteors brighter than third magnitude only, but tentatively it may be taken as giving the type of distribution. Now the relative rates of energy radiated for a difference in magnitude j' is given by $E_{(j+j')}/E_j = (\sqrt[6]{100})^{j'}$. Thus, taking the frequency of our standard first magnitude meteor as given, a table may be constructed giving frequencies, or what is more useful, densities of meteors corresponding to different lower mass limits. Table III has been calculated in this way, using the Öpik value of the constant, 4.

TABLE III

FREQUENCY AND DENSITY DISTRIBUTIONS FOR VARIOUS LOWER MASS LIMITS

Mass (Maris) in gms.	Approx. radius (cm.)	Visual magnitude	Frequency of meteors above this mass (Öpik distribution, 1st Mag. = 1)	Mean volume containing 1 meteor above mass limit, in sporadic dist. (cu.kms.)
$6{\cdot}25 \times 10$	1·14	− 8	$1/(2{\cdot}6 \times 10^6)$	$5{\cdot}7 \times 10^{14}$
6·25	0·53	− 5	1/4096	$9{\cdot}0 \times 10^{12}$
$6{\cdot}25 \times 1/10$	0·25	− 2	1/64	$1{\cdot}4 \times 10^{11}$
$6{\cdot}25 \times 10^{-2}$	0·12	+ 1	1	$2{\cdot}2 \times 10^{9}$
$6{\cdot}25 \times 10^{-3}$	0·05	+ 4	64	$3{\cdot}4 \times 10^{7}$
$6{\cdot}25 \times 10^{-4}$	0·02	+ 7	4,096	$5{\cdot}4 \times 10^{5}$
$6{\cdot}25 \times 10^{-5}$	0·01	+10	$2{\cdot}6 \times 10^{5}$	$8{\cdot}5 \times 10^{2}$

From normal observations, such highly approximate calculations are as far as one can go. Very recently, however, attempts have been made to calculate the distribution in mass from the distribution in intensities of the

echoes on the radar apparatus. This intensity is given as $I = ka^2\lambda^3 P_0 G^2/R^3$ where a, the number of ions formed per cm. of path, is proportional to $1/r$, r the radius of the meteor's (assumed circular) cross-section, λ is the fundamental wavelength used, P_0 is the peak power of the transmitter, G the aerial gain, R the range, and k a constant calculated from the geometry of the meteor-observer system.* (The law "a" proportional to "$1/r$" is obtained from some unpublished calculations by Horlofson.) The above is as given by Lovell, Banwell and Clegg,[14] who from observations of two showers give the frequency of meteors of radius r, $N \propto 1/r^3$. Statistically, therefore, $N \propto 1/(\text{volume})$, or, assuming constant density, $N \propto 1/\text{mass}$. This law of distribution may thus be taken also as quasi-observational. It gives rise, by simple integration, to the law that the number of meteors between masses m_1 and m_2, $m_1 N m_2 \propto 1on\ (m_2 \div m_1)$. This has the disadvantage that it approaches infinity as m_2 approaches infinity. Clearly the law is not applicable over such large a range. The actual measurements appear to have covered a radius-range 0·01–0·03 cms. only, i.e. a mass-range from 10^{-5} to 10^{-3} gms. Although this law is applicable only for the very smallest meteors, and has thus not been included in Table III, it does indicate that observations by radar methods, combined with more comprehensive physical theories to cover the new observations, may in the future give very definite information of mass distribution.

Meteorites

A few brief words may be said on meteorites. They show the duality of chemical composition already remarked upon. Wylie[15] has estimated that, for the whole Earth, 11,000 meteorites fall each year, or 1 meteor every 9,000 years for a surface area on the Earth of 50 square miles. The rate of increase of the mass of the Earth due to meteoric accretion is 1 gm./square mile/year or 2×10^6 kilograms/year. The Earth's radius would increase by 1 inch every 2×10^{10} years (note: estimated age of the Earth $= 5 \cdot 10^9$ years).

Other Meteoric Matter

So far we have dealt only with meteoric matter which at some time enters the Earth's atmosphere and is rendered visible by the transformation of its relative kinetic energy into radiant energy. We have no direct knowledge of any other meteoric matter, but it is worth while examining what indirect information we have. We would expect to detect such "extra-terrestrial" material by either (i) reflection of light, or (ii) absorption of light.

We have seen that a close connection must exist between comets and meteor swarms. Although the exact nature of this connection is still in doubt, there seems little objection to assuming that a comet is an aggregation of meteoric material held together by its mutual gravitation (unless ejected, by pressure of solar radiation, in the form of a cometary tail). A comet, although denser than any part of a meteor swarm, is still so diffuse that no diminution of light has with certainty been detected in the light of a star even when the nucleus of a comet passes over it. The suggestion has recently been made[16] that the nucleus of a comet consists of an aggregation of pieces of matter of

* $k = 8 \cdot 88 \times 10^{-28}$ *for a radio beam directed perpendicular to the meteor track.*

size the order of 1/10 kms., separated by very small distances. Such matter would constitute a "missing link" in size between the larger meteorites and the smaller asteroids. The comet's coma and tails consist of particles much smaller than normal meteors.* Thus we may consider comets as meteoric matter, and expect a meteor swarm in the path of a major comet. It should be noted, however, that the stream may be perturbed so that it no longer follows its associated comet (e.g. the orbits of the η Aquarids and of Halley's Comet are 4 million miles apart in the vicinity of the Earth, but they appear to be parallel); also a meteor stream is more durable than a comet; thus meteor streams may, and do, exist with no associated comet. It would be difficult to obtain knowledge of such streams, but in this connection the suggestion recently made by Gregory, and examined by Davidson,[17] may be of importance, namely, that the brightening of comets, which appears to be neither regular nor periodic, may be due to the passage of the comet through a meteor stream. There is thus the highly speculative possibility of detecting such streams by the analysis of the light-curves of comets.

The asteroids, which we detect by reflected sunlight, may be considered meteoric matter. With the single exception of Vesta, they are invisible to the naked eye, and the vast majority of them have been discovered photographically. The number known at the moment is about 2,000. The average rate of discovery in the years before the war was one per day (Wolf at Heidelberg has over 500 such discoveries to his credit). They revolve in orbits which, in general, lie between Mars and Jupiter, and they differ from the planets in their much greater variety of orbits. While the planets' orbits are nearly circular, with small inclination to the ecliptic, asteroids may have highly eccentric, highly inclined orbits (e.g. Hildago $\eta = 0{\cdot}65$, $i = 43°$). In size, estimated from their brightness, since few show detectable telescopic discs, the majority are less than 70 miles in diameter, about a dozen are known to exceed 100 miles, and some are as small as 1 mile (e.g., Apollo). It has been estimated that there are 100,000 asteroids brighter than stellar magnitude 20, with a combined mass = 1/2000 that of the Earth. No asteroid is large enough to retain an atmosphere. Fifteen per cent. of known asteroids can be grouped into "families," members of which have similar mean distances from the Sun, and similar inclinations to the plane of Jupiter's orbit (Jupiter exerting great perturbing forces on the asteroids). There is indication that two of these families at least had similar orbits 3 million years ago, and they may have come from the tidal disruption by Jupiter of a single body.

In the Zodiacal Light we have indication of the existence of reflecting matter. The Zodiacal Light is a faint band of light stretching along the ecliptic to about 70° from the Sun to the East and West. It is best seen in the tropics, when it has at times been traced, as the Zodiacal Band, right round the ecliptic. The light is believed to be due to the scattering of sunlight by material which has a strong preference for a plane not much inclined to the plane of the ecliptic, and which is more concentrated within the Earth's orbit than outside it. Spectroscopically, the Zodiacal Light is in general similar to sunlight, and hence the scattering (or, more strictly, reflecting) agents are probably large

* *Diameter about* 10^{-4} *cm.*

compared with the wavelength of light (i.e. not molecular), but this conclusion is a little uncertain because of the uncertainties of the spectroscopic observations. Russell estimates that the Zodiacal Light can be accounted for if the reflecting material within the Earth's orbit be composed of particles a millimetre in diameter, separated by distances of the order of 10 kilometres. So diffuse a distribution of matter could not appreciably affect the motion of the planets or comets, and it is visible solely on account of its great volume. An estimate of the distribution of matter in the Zodiacal Light has been made recently by Allen,[18] where he interprets spectrum observations of the Corona supposing it due to reflection and diffraction of sunlight by small particles, and extends his calculations to the Zodiacal Light. He finds the best fit with observation by the distribution $N(r) = N_e(D/r)$, where "r" is the radial distance of the particles from the Sun, "D" the distance of the Earth from the Sun, N_e the number of particles per c.c. at the Earth's distance, and $N(r)$ the number per c.c. at radial distance r. He finds the most reasonable values of the constants to take are $N_e = 3 \cdot 10^{-15}$ cms.$^{-3}$, and the radius of the particles (assumed constant) $= 10^{-3}$ cms.

Before leaving the solar system, mention should be made of Saturn's Rings, which are composed of large numbers of meteoric particles circling Saturn in orbits like thousands of small moons. That the rings are discontinuous in structure is shown by gravitational theory, a continuous structure being dynamically impossible. Spectrum observations have proved the matter by showing that the orbital angular velocity of the inner parts of the rings is greater than the outer parts. Further proof is the visibility of the rings by reflected sunlight no matter what the inclinations of the rings to the Sun and Earth. These rings were probably formed by the disruption of a Saturnian satellite by tidal forces of Saturn and the other satellites.

Outside the solar system, we have evidence of large-scale distributions of matter in the opaque clouds, clearly visible in the Milky Way as apparent absence of stars over certain regions. There are also the "reflection nebulae," such as the Orion Nebula, or the Pleiades Nebulosity, which shine by light reflected from stars in their vicinity. It is reasonable to suppose that the light of all the bright galactic nebulae is at least in part the reflected light of their associated stars. (When the star is hot enough to stimulate appreciable emission of light from the nebulous material itself, the reflected light, spread over the whole spectrum, may be inconspicuous compared with the emission light concentrated in a few bright lines.) It is important to know whether the light of these "reflection nebulae" is scattered by particles relatively large compared with the wavelength of light, or not (i.e. particular or molecular). In the first case the nebular light should have the same colour as the light of the associated star, whereas in the second it should be bluer. Struve, Elvey and Roach solved the problem in 1936 by photographing, at the McDonald Observatory, the region in Scorpio and Ophiuchus (in particular, the region round the star Antares) where excellent examples of these reflection nebulae occur, in light of various colours (by using filters). The general spectral distribution of the nebula light was found to conform to that of the neighbouring star (e.g. the glow round the red star Antares is more conspicuous using a yellow

filter than without it, while the nebulosity around the blue star δ Scorpii is more conspicuous without the yellow filter). Thus the material of this reflection nebula at least is composed of particles large compared with the wavelength of light.

Lastly, the presence of "interstellar lines" in the spectra of many stars (detected in spectroscopic binaries as they do not partake in the periodic Doppler shifts of the stars' spectral lines, nor are they "telluric" lines due to the Earth's atmosphere), and also the general reddening of light from distant stars indicate wide distribution in the galaxy of diffuse absorbing matter. The exact nature of this interstellar material is not known with certainty, but it is probably of a molecular nature.

Points of Special Interest Relevant to Interplanetary Travel

The first important point to note is that it is quite impossible to plot individual meteors and plan to avoid them. At best, one must rely upon chance collisons not occurring. However, certain precautions can be taken. Since the chance of collision is greatest in the densest distributions, it is advisable to avoid, as far as is possible, the vicinities of known meteor streams, and the orbits of known major periodic comets. The plane of the ecliptic appears to have an abnormally high density of matter, and might also be avoided, although the particles composing it are probably well below the critical mass limit.

The probability of a collision occurring in a given density of meteors obviously depends upon the relative velocity of the space-ship and the meteors, and also on the size of the space-ship. A vessel of cross-section A travelling with a velocity v relative to the system of meteors whose density is one particle in every volume V, sweeps out in unit time a volume $S = Av$. The ratio of volumes S/V gives a measure of the probability of a collision. (With such highly approximate calculations mathematical rigour is spurious.) Below a certain critical mass a meteor colliding with the space-ship will do no damage, and can be ignored. This critical mass depends upon the construction of the vessel, but would probably be of the order 10^{-3} gm. In Table IV the time

TABLE IV

PROBABLE-COLLISION TIMES

(in days, for space-ship of cross-section 500 square metres, velocity relative to meteor system 1 km./sec.)

Lower mass limit (gms.)	10	1	1/10	10^{-2}	10^{-3}	10^{-4}	10^{-5}
Type of distribution:							
Sporadic	1×10^{13}	2×10^{11}	3×10^{9}	4×10^{7}	7×10^{5}	1×10^{4}	200
Minor shower ..	5×10^{12}	1×10^{11}	2×10^{9}	2×10^{7}	3×10^{5}	5×10^{3}	100
Major shower ..	2×10^{12}	3×10^{10}	7×10^{8}	7×10^{6}	1×10^{5}	2,000	30
Major shower (prolific)	4×10^{11}	6×10^{9}	1×10^{8}	1×10^{6}	2×10^{4}	400	6
Major shower (very prolific)	8×10^{10}	1×10^{9}	2×10^{7}	2×10^{5}	4×10^{3}	80	1

taken by a standard space-ship (whose cross-section is 500 square meters, and its relative velocity 1 km./sec.) to sweep out a volume = V is given as the

"probable-collision" time. It is tabulated for various lower limits of mass, and for various types of distribution. Under such tabulated conditions, it is probable that a dangerous collision will occur in the appropriate "probable-collision" time.

It would appear, at the moment, to be an impossibility to design a practical space-ship which is adequately protected against meteors and meteorites. Until such time as protection can be secured, or a means of fending off or avoiding these bodies is devised, collision with meteoric matter will always be a risk of interplanetary travel. Possibly this risk has been exaggerated in the past, but it is nevertheless a real risk, and one that deserves the closest consideration of a would-be space traveller.

Recently, Whipple has examined the possibility of surrounding the space-ship with a fairly thin layer of protective material, separated from the surface of the vessel, which would dissipate the energy of a meteor without causing damage. He concludes that such a meteor "cushion" would afford definite protection against meteors of small mass that might otherwise prove dangerous. Such methods of minimising the dangers of meteoric collisions should be looked for.

In conclusion, if a conclusion can be drawn from such perforce scant data, it would seem that the risks involved are not too great, and that the space traveller will not be committing certain suicide from a meteor collision. No risk, however great, could deter mankind from ever widening its scope of knowledge and activity, but it is comforting to know that, fraught with dangers as it will be, the first stage in the conquest of space will, in this respect at least, stand a healthy chance of succeeding.

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LAGRANGE'S EQUATIONS FOR A SYSTEM OF VARIABLE MASS

By DEREK F. LAWDEN, M.A.

A rocket is an example of a general dynamical system of variable mass, and a complete discussion of its motion must consequently be based upon the equations of motion of such a system. The modifications which have to be made to Lagrange's equations before they can be applied to a system of variable mass, holonomic and possessing n degrees of freedom, will accordingly be explained. It is assumed that the reader is familiar with the process whereby Lagrange's equations are derived from the equation of virtual work; the equations are usually quoted in the form

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} = Q_i \ (i = 1, 2, \ldots n)$$

where the q_i are the generalised co-ordinates of the system, Q_i are the generalised components of the active forces, i.e. the forces which do work in a virtual displacement of the system consistent with the constraints, and T is the kinetic energy.

As regards notation, Σ will denote summation with respect to particles of the system, all other summation signs with respect to suffixes being omitted according to the summation convention of the tensor calculus.

Let m be a typical particle of the system, $\mathbf{r}$ be its position vector with respect to a fixed frame of reference.

Suppose m to be absorbing material from its surroundings at a rate dm/dt, $\mathbf{u}$ being the velocity vector of the material with respect to the fixed frame immediately before assimilation. For example, m may be a raindrop descending through a cloud of water vapour from which moisture is condensing on to the drop; $\mathbf{u}$ is then the velocity of the water vapour in the immediate vicinity of the drop.

The "rocket" equation for a particle gives

$$\frac{d}{dt}(m\dot{\mathbf{r}}) = \mathbf{F} + \mathbf{u}\frac{dm}{dt} \qquad \text{(i)}$$

where $\mathbf{F}$ is the vector specifying the force acting on m.

It should be noted that the term $\mathbf{u}\,dm/dt$ accounts only for the succession of infinitesimal impulses to which m is subjected as it absorbs material. This term does not account for any other influence this material may have on m, such as, for example, electrical attractions or repulsions between m and particles of the atmosphere from which it is absorbing material. Any such additional interactions between the atmosphere and the particle m must be allowed for by inclusion in $\mathbf{F}$. In order to convince the reader of the importance of this point, consider a complex system such as a rocket. Material is being ejected from the system rather than being absorbed, but mathematically this is merely a question of sign. Two specifications of this dynamical system will be considered:—

(*a*) The system may be taken to include, at any given time, everything

but particles of burnt fuel. The gases in the combustion chamber are therefore excluded from the system by this definition.

(*b*) The gases in the combustion chamber and venturi may be included in the system, only particles of exhaust gases which have left the venturi being excluded.

If it is decided to adopt point of view (*a*), the terms in the equations of motion corresponding to $\mathbf{u}\, dm/dt$ of (i) for a typical particle of the system will allow for the immediate effects of burning fuel in the combustion chamber, and since immediately subsequent to combustion the exhaust particles have a more or less random motion these effects can be expected to be small. It follows that these terms do not correspond to the main thrust effect of the motor. This thrust effect is, in fact, an additional interaction of the exhaust particles with the system and must be included in the terms corresponding to **F**. The random motion already referred to is converted into a steady streaming towards and out from the venturi by the design of these and of the combustion chamber. This leads to a non-uniform distribution of pressure over the walls of the combustion chamber and a resultant thrust on the rocket which must be included in **F**.

If, however, we decide to adopt the alternative point of view, so that the system is supposed to extend to the venturi, the particles ejected may be supposed, quite reasonably, to have no further influence over the rocket's motion and so the terms corresponding to $\mathbf{u}\, d\mathbf{M}/dt$ in the equations of motion must now include the main thrust and there are no secondary effects to be included in **F**. The pressure of the exhaust gases on the combustion chamber walls are now to be regarded as internal forces and are cancelled by the reaction of the chamber walls on the gases.

It might seem from the above that the second approach possesses all the advantages, but it must be borne in mind, however, that the dynamical system in this case is by no means as simple as it is in the first case. Including, as it does, the exhaust gases, it possesses theoretically an infinity of degrees of freedom, whereas by defining the system as in (*a*) we have restricted ourselves to a system which may usually be considered as having a finite number of such degrees. If, therefore, the dynamical effects of the gases in the combustion chamber and venturi may be approximated with sufficient accuracy and no great difficulty (e.g. as a constant thrust along the axis of the machine), it is certainly advisable to approach the problem as in (*a*). Such approximations will almost certainly neglect, amongst others, effects due to the rotational inertia of the gases in the combustion chamber. Such an effect will be negligibly small of course, and would only be insisted upon in an academic problem.

By assuming some simple steady streaming of exhaust gases within the rocket it is possible to reduce effectively the number of degrees of freedom of the system, defined as in (*b*), to a small finite number and then this approach to the problem becomes practicable.

Some time has been spent on the question of the manner in which we are to assign boundaries in order that a rocket may be regarded as a typical dynamical system of variable mass, since it is felt that consideration of this

question ventilates a number of difficulties associated with the dynamics of the rocket, the solutions to which are not always crystal clear even to the initiated. As a final point in this connection it may be noted that, by indefinite extension of the boundary, the system may be taken to include all particles of the exhaust gases both within and without the rocket. The system is then of constant mass and the ordinary dynamical equations apply to it.

From (i) by summation with respect to all particles m of the system

$$\frac{d}{dt}(\Sigma m\dot{\boldsymbol{r}}) = \Sigma\mathbf{F} + \Sigma\boldsymbol{u}\frac{dm}{dt}$$

Now $\Sigma m\boldsymbol{r}$ is $\boldsymbol{p}$, the linear momentum vector of the system at any moment. It is proved in the textbooks that

$$\boldsymbol{p} = \mathrm{M}\dot{\boldsymbol{x}}$$

where M is the mass of the system and $\dot{\boldsymbol{x}}$ is the velocity of its C.M. It must be noted here that when calculating $\dot{\boldsymbol{x}}$ in any particular case only those particles which are considered as within the system at the moment we have in mind make a contribution. Thus $\dot{\boldsymbol{x}}$ is not the velocity of the C.M. of the system of variable mass but the velocity of the C.M. of the system of constant mass formed by those particles within the system at the time being considered. The velocity of the C.M. of the system of variable mass is compounded of two components (i) $\dot{\boldsymbol{x}}$ and (ii) the motion of the C.M. relative to the system itself, due to the aggregation of new material. If, for example, a rocket is being translated along its axis at a velocity $\mathbf{V}$, then $\dot{\boldsymbol{x}} = \mathbf{V}$, whereas the velocity of the C.M. of the rocket is $(\mathbf{V} + \mathbf{U})$, where $\mathbf{U}$ is the velocity atwhich the C.M. is moving towards the nose, relative to the rocket, as the fuel is consumed. $\mathbf{U}$ will, of course, in this case be small.

$\Sigma\mathbf{F}$ will reduce, as always, to the external forces, the internal reactions cancelling.

Finally it is easily seen that

$$\Sigma\frac{dm}{dt}\boldsymbol{u} = \frac{dm}{dt}\boldsymbol{u}_0$$

where $\boldsymbol{u}_0$ is defined as follows.

Consider the particles which become incorporated in the system in time δt. Let $\boldsymbol{u}'$ be the velocity of their C.M. immediately before assimilation. Then $\boldsymbol{u}' \to \boldsymbol{u}_0$ as $\delta t \to 0$.

The equation of linear momentum may therefore be written

$$\frac{d}{dt}(\mathrm{M}\dot{\boldsymbol{x}}) = \Sigma\mathbf{F} + \frac{d\mathrm{M}}{dt}\boldsymbol{u}_0 \qquad \ldots\ \ldots\ \ldots \qquad \text{(ii)}$$

Equation (ii) is in convenient form for the solution of problems in which particles are being deposited on to a system from an atmosphere, since $\boldsymbol{u}_0$ is then usually easy to calculate. In "rocket" problems, however, since the exhaust gases partake of the rocket's own motion, $\boldsymbol{u}_0$, being relative to fixed axes, is not so easily obtained and it is better to modify the equation of linear momentum by observing that (i) may be written

$$m\ddot{\mathbf{r}} = \mathbf{F} + \boldsymbol{v}\frac{dm}{dt}$$

where $\boldsymbol{v} = (\boldsymbol{u} - \dot{\boldsymbol{r}})$ is the velocity of the atmosphere relative to the particle itself. Then summing over all particles

$$\Sigma m\ddot{\mathbf{r}} = \Sigma\mathbf{F} + \Sigma\boldsymbol{v}\frac{dm}{dt}$$

Now $$\Sigma m\ddot{\mathbf{r}} = \mathrm{M}\ddot{\mathbf{x}}$$

where $\boldsymbol{x}$ is the acceleration of the C.M. of all particles within the system at the moment under consideration. It is again necessary to give warning that

$$\frac{d}{dt}(\dot{\mathbf{x}}) \neq$$

for the same reasons that

$$\frac{d}{dt}(\mathbf{x}) \neq \dot{\mathbf{x}}$$

as was explained above.

Also, $\Sigma\boldsymbol{v}\, dm/dt$ may be reduced to $\boldsymbol{v}_0\, d\mathrm{M}/dt$ as before, $\boldsymbol{v}_0$ being the velocity of the C.M. of those particles of the atmosphere which attach themselves to the system in the small time δt, in their motion relative to the system. $\boldsymbol{v}_0$ is easily calculable in most rocket problems.

Hence the equation of linear momentum is

$$\mathrm{M}\ddot{\mathbf{x}} = \Sigma\mathbf{F} + \boldsymbol{v}_0\frac{d\mathrm{M}}{dt} \quad .. \quad .. \quad .. \quad .. \quad \text{(iii)}$$

Provided that the motion of the C.M. about and relative to the system may be neglected, $\ddot{\mathbf{x}}$ is the acceleration of the C.M. of the system. In all elementary treatments (iii) is quoted as the fundamental equation and this assumption is made.

To obtain Lagrange's equations, suppose $\boldsymbol{r}$ to depend explicitly on the n generalised co-ordinates q_i $(i = 1, 2, \ldots n)$ and the time t. If then $\delta\boldsymbol{r}$ is a virtual displacement of m, consistent with the constraints at time t, the equation of virtual work for the system is

$$\Sigma\frac{d}{dt}(m\dot{\boldsymbol{r}}) \,.\, \delta\boldsymbol{r} = \Sigma\mathbf{F} \,.\, \delta\boldsymbol{r} + \Sigma\boldsymbol{u}\frac{dm}{dt} \,.\, \delta\boldsymbol{r}$$

Now $$\delta\boldsymbol{r} = \frac{\partial\boldsymbol{r}}{\partial q_i}\,\delta q_i$$

and the equation may be reduced exactly as for a system of invariant mass to

$$\left\{\frac{d}{dt}\left(\frac{\partial \mathrm{T}}{\partial\dot{q}_i}\right) - \frac{\partial \mathrm{T}}{\partial q_i}\right\}\delta q_i = \mathrm{Q}_i\delta q_i + \Sigma\frac{dm}{dt}\,u \,.\, \delta\boldsymbol{r}$$

and since the system is holonomic, i.e. the δq_i are always independent,

$$\frac{d}{dt}\left(\frac{\partial \mathrm{T}}{\partial\dot{q}_i}\right) - \frac{\partial \mathrm{T}}{\partial q_i} = \mathrm{Q}_i + \mathrm{Q}_i' \quad .. \quad .. \quad \text{(iv)}$$

where Q_i' is the coefficient of δq_i in $\Sigma dm/dt\, u \,.\, \delta r$. To calculate Q_i', at each point of the system at which material is being assimilated imagine a force $dm/dt\, u$ to be applied. The work done by such a system of forces during a displacement $\delta q_i (i = 1, 2, \ldots n)$ is $Q_i' \delta q_i$ and from this linear form Q_i' may be picked out.

If $u \equiv 0$, i.e. material is being absorbed from a stationary atmosphere, $Q_i' \equiv 0$ and Lagrange's equation in its normal form becomes applicable.

An alternative form of (i) is

$$m\ddot{r} = F + v\frac{dm}{dt}$$

where v is now the velocity of the atmosphere in the vicinity of m, relative to m. This equation leads to an alternative form of the equation of virtual work, viz.:

$$\Sigma m\ddot{r} \,.\, \delta r = \Sigma F \,.\, \delta r + \Sigma \frac{dm}{dt} u \,.\, \delta r$$

which in turn leads to another form of Lagrange's equations—

$$\left(\frac{d}{dt}\right)'\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} = Q_i + Q_i'' \quad \ldots \quad \ldots \quad \ldots \quad \text{(v)}$$

$(d/dt)'$ being interpreted as meaning that when differentiating $\partial T/\partial \dot{q}_i$ with respect to t, the mass of each particle is to be regarded as constant. Q_i'' is calculated in a manner similar to that used in calculating Q_i', the relative velocities of the particles being absorbed now being taken into account instead of their velocities relative to the fixed frame.

It will be found that form (iv) is suitable for most systems into which matter is being assimilated and form (v) is suitable for systems from which matter is being ejected.

Applications of these equations to specific problems will form the subject-matter of a later article.

ABSTRACTS

Journal of the Aeronautical Sciences, March, 1947

Almost every issue of this periodical these days contains some article of interest on supersonic aerodynamics; the March, 1947 issue has two:—

Aerodynamic Centre and Centre of Pressure of an Airfoil at Supersonic Speeds, by Prof. H. W. Sibert (University of Colorado), and

Some Ballistic Contributions to Aerodynamics, by A. C. Charters, of the U.S. Army Ordnance Department, Aberdeen Proving Ground.

The latter, dealing with the resistance of projectile shapes at very high speeds, is particularly interesting. The author states that experimental results are in agreement with the theoretical predictions of von Kármán and Moore, and of Taylor and Maccoll. He also claims that small scale model tests will be adequate for predicting full scale performance. The article quotes many calculated and experimental figures, also a number of useful references, and concludes with a discussion of the optimum body shape for supersonic speeds.

It is thought that a conical nose (semi-vertex angle = 4°) is near to the

best possible; an ogival nose might be slightly better, but only to the extent of a few per cent. Even at a Mach No. of 5·0, it is worthwhile to use a "boat-tail," i.e. a tapered tail rather than a square-based one. A "boat-tail" will save at least 24 per cent. of drag even for this high speed, and more at lower speeds, although previously ballistics experts have often asserted that the practice is not worthwhile at high supersonic speeds. It is probable that a truly conical tail would be rather better than an ogival one, and a semi-vertex angle of 7° 15′ is considered a good choice.

At a Mach No. of 1·5, skin friction and base pressure drag are nearly equal and together account for 85 per cent. of the drag; at a Mach No. of 5·0, however, skin friction alone accounts for 75 per cent. Taking a body shape of the type described above and having a maximum diameter of 50 ins., the power required to propel it at a Mach No. of 1·5 would be 21,243 h.p. at sea level, but only 2,263 h.p. at 50,000 feet.

Discovery of a Daylight Meteor Shower

B.A.A. Circular, No. 282 (1947, June 26) gives details of the discovery by Dr. A. C. B. Lovell and the Jodrell Bank (Manchester) Radar Research Group of a major meteor shower occurring in daylight. They used directional aerials of beam width 12° to examine individual radiants, and found radiant points by the (unpublished) method of Clegg. In the investigation of the η Aquarid radiant, which began on May 1, 1947, it became obvious that the visible stream was merely the beginning of an extremely active belt stretching towards the sun. Preliminary analysis gives the approximate positions of the new radiant as:—

May	7 ..	R.A. 20°	Dec. ..	+ 20° to 25°
June	22 ..	70°		20° to 25°

The Aquarius end of the belt disappeared, as expected, in mid-May, but the new radiant, in Pisces, continued with extraordinary intensity. The shower had (until June 22, when information was disseminated) been active for 53 days, and showed no sign of ending. The hourly rate fluctuated between extremes of 20 and 80 (the surprising nature of which is seen by comparing the major Geminid shower of December, 1946, which gave hourly rates of between 10 and 25 with the same equipment). The shower radiant transit occurred at about 1,000 U.T. and thus could never be detected by visual observation. The discovery marks an important advance in the study of meteors, and indicates the power of the radar method of observation. We await with interest further application of the method, and the new knowledge it will give us.

Measurements from V.2 Rockets

At a joint meeting of the Institute of Aeronautical Sciences and the American Meteorological Society in New York on 30th January, Mr. Nolan R. Best gave an extremely interesting talk on the use of V.2 rockets for measuring temperature and pressure in the upper atmosphere. Instrumental difficulties are caused by the high speed of the rocket (almost Mach No. 5), but special gauges have been devised to overcome these. A radio telemetering system has been developed

so that continuous readings from many instruments can be broadcast during flight. Despite the fact that much desirable aerodynamical data are not available for V.2, it has been necessary to proceed with the experimental programme at the high rate of firing which the U.S. Army has set owing to the deterioration of the German V.2, and also because "better rockets will soon be available."

It is hoped eventually to provide gauges which will cover the whole range in pressures encountered during flight, i.e. from 760 mm. Hg to about 10^{-7} mm. —a range of nearly ten thousand million. So far only part of this range has been covered. Mechanical gauges of the bellows type have been used for atmospheric pressure down to about 10 mm. Hg, after which Pirani gauges take over. These are simply small electric lamps open to the atmosphere. The cooling effect of the surrounding gas varies the resistance of the filament, enabling pressure readings to be taken down to a few thousandths of a mm. Hg. The gauges are mounted in heavy brass shields to keep the temperature substantially constant during flight.

The mounting of the gauges on the rocket raises considerable problems, owing to the pressure variations over the surface of such an irregular shape. German wind tunnel figures have been employed to select what seem the most suitable places, and the results have been checked in actual flight up to a Mach No. of 2 by simultaneous readings by weather balloons. The agreement with the theoretical calculations is fairly good. The gauges, however, would not operate in regions where the mean free path of the gas molecules is comparable to their own dimensions.

Though temperature gauges have been used, direct measurements are not possible above altitudes of 50 Kms. owing to the small heat transfer at the low air densities. Indirect estimates can, however, be obtained from the pressure readings. Tentative conclusions show that the temperature at 65 kms. is about 300° K. (27° C.) and at 78 Kms., 220° K. (— 53° C.). Thereafter the temperature is believed to rise again, but further readings are needed to confirm this.

The lowest pressures so far recorded are about a hundred thousandth of an atmosphere at an altitude of 90 Kms. At the heights of twice this, which have already been reached, the pressure must be something like a million millionth of an atmosphere and it seems unlikely that it can be measured by direct instrumental means.

ACKNOWLEDGMENT.—The Librarian begs to acknowledge with thanks receipt of a copy of *Ballistics of the Future* kindly donated by Mr. Walter H. Gillings.

URGENTLY WANTED.—*Jnl. R.Ae.S.*, July, 1946 (or reprint Art. Perring), *Mechanical Engineering* (U.S.A.), March and July, 1946; *Aeronautics*, May, 1946; *Acrft. Eng.*, Februaty, 1942, and June, 1945; *The Engineer*, October 19, 1945, and *Flight*, November 8, 1945; *Practical Mechanics*, October, 1946; B.I.S. *Jnls* prior to 1938. Interested in other prints or documentary material relating to astronautics in all countries. Will purchase or exchange. Ir. G. de Koningh, van der Brugghenstraat 18, Delft, Holland.

RESEARCH PAPERS

The following is a further selected list of recent papers, lectures and other publications of particular interest to those engaged on research.

1. *Metallurgical Aspects of Metal and Metal-Ceramic Bodies for use in High Temperatures in Jet- and Rocket-Engine Services.* E. A. Kachik *et al.* Memo. Report No. 5, School of Mineral Industries, Pennsylvania State College, Department of Commerce Publication No. PB 33139.
2. *Rocket Fuels.* A. Kylberg and L. G. Sundblad. Tek. Tid. (Sweden), 1946, pp. 1,117-24. A review with 11 references.
3. *Unique features built into XS–I; Bell rocket-powered flying supersonic laboratory.* Aviation News, December 2, 1946, Vol. 6, p. 12, illustrated.
4. *First XS–I powered flight opens drive to supersonic speeds.* Aviation News, December 16, 1946, Vol. 6, p. 9, illustrated.
5. *U.S. Rocket engine flight test points way to new developments.* Aviation News, December 23, 1946, Vol. 6, pp. 8-9, illustrated.
6. *Rockets in battle.* W. Ley. Technology Review, December, 1946, Vol. 49, pp. 95-100, illustrated.
7. *Midget Electronic Equipment.* Electronics, February, 1947, Vol. 20, No. 2, pp. 84-9, illustrated. Using sub-miniature tubes and comparably small circuit components, one can build electronic equipment small enough to be comfortably carried in a suit pocket. Design data, types of components that are available, and possible applications are presented.
8. *Assisted Take-off.* Inter Avia, February 20, 1947, No. 1,272, p. 3. Table illustrating increase in permitted take-off weight of Douglas D.C.–4 four-engined commercial transport from various airfields using four Aerojet A.T.O. units (17–47 per cent. increase).
   Inter Avia, February 22, 1947, No. 1,272, p. 2. Aerojets have produced a new high thrust long duration A.T.O. rocket for use with multi-engined jet bombers. Preliminary tests to be carried out on XB–45.
   Inter Avia, March 27, 1947, p. 1. Svenska Aeroplan J21R jet fighter has undergone A.T.O. tests with rockets developed by Bofors A.B. Take-off run has been cut by one-third.
9. *New Supersonic Aircraft.* Inter Avia, February 22, 1947, No. 1,272, p. 2. Northrops building new supersonic aircraft, the XS–4. No details released but it may be an all-wing type fitted with a rocket engine. This would be in line with the current Northrop work on rocket units.
10. *Guided Missiles Range.* Inter Avia, February 22, 1947, No. 1,272, p. 6. The New Permanent Guided Missiles Testing Range of the U.S. is to be at Pt. Magu, South California and will stretch 5,000 miles across the Pacific.
11. *L'Exploration des hautes altitudes par les fusées.* M. Déribéré. La Nature March 1, 1947, No. 3,131, pp. 73–6, illustrated.

12. *Telemetering from V.2 Rockets.* V. L. Heeren *et al.* Part I, Electronics, March, 1947, Vol. 20, No. 3, pp. 100–5, illustrated. Time-modulated pulse system in nose of rocket samples successively up to 23 instruments and transmits readings to ground station over 1,000 Mc/s radio link, using 2C43 disk-seal tube as oscillator. Airborne circuits only given in Part I. Part II, Electronics, April, 1947, Vol. 20, No. 4, pp. 124–7, illustrated. Output of 1,000 Mc/s receiver in ground station of time-modulated pulse telemetering system contains trains of pulses spaced according to instrument readings in rocket. Circuits for de-coding these pulses into individual voltages for recording are given.

The following references numbered 13 to 16 inclusive are from Army Ordnance, March–April, 1947, Vol. 31, No. 161.

13. *Into the blue at White Sands.* Pp. 425–8. Series of photographs of preparation and firings at White Sands.
14. *Rockets in the Desert.* Lieut. Col. H. Rivkins. Pp. 429–31, illustrated. Review of V.2 firings at White Sands, with analytical table of result so far obtained.
15. *Jet Propulsion.* H. M. Shaw. Pp. 435–7, illustrated. Descriptions of the working of pulse-jets, ram-jets and rockets with details of examples (V.1, V.2, W.A.C. series).
16. *Guided Missiles.* G. White. Pp. 438–40, illustrated. Discussion of devices for controlling long-range rockets.
17. *Gaging Rocket-Engine Forces and Flows.* G. P. Sutton. Aviation, April, 1947, Vol. 46, No. 4, Pp. 46–8, illustrated. A non-detailed description of the characteristics of the instruments required for rocket-engine performance; covers thrust, propellant flows, pressures, temperatures, exhaust gas velocity, flame composition, size and shape.
18. *On the Steady Flow of a Gas Through a Tube with Heat Exchange or Chemical Reaction.* Chambré, Paul and Lin, C. C. J. of the Aero. Sc., October, 1946, Vol. 13, No. 10, pp. 537–42, illustrated, references.
19. *The Radio Detection of Meteor Trails and Allied Phenomena.* Sir Edward Appleton, F.R.S., and R. Naismith, *Proc. Phys. Soc.*, 59, 461, May 1947. A useful summary of work done by pioneers in the subject, giving some real idea of the problems involved, methods of tackling them and conclusions so far reached.

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REVIEWS

Gas Turbines and Jet Propulsion for Aircraft

(By G. Geoffrey Smith, Editorial Director of *Flight* and *Aircraft Production*. 12s. 6d. net, 246 pp., over 200 illustrations (demy 8vo.). Fourth Edition.)

The publication of yet another edition of this well-known book is a timely reminder of the increasing importance of turbine power units, in both military and civil aircraft development.

The 21 chapters of this edition will meet a definite need for concise and comprehensive information. In addition to dealing with fundamental principles, construction, operation, and the testing and maintenance of power units, descriptions are given of existing British, American and German units and aircraft, together with details of projected future types.

British Thought, 1947

(Published by the Gresham Press, Inc., New York.)

(Obtainable by special order from H. K. Lewis & Co., Ltd., 136, Gower Street, W.C.1.)

We were very interested to receive a copy of this book, which collects together a number of outstanding essays and original papers published in this country during 1946.

Contributions range from science to economics and politics, and it is apparent that considerable care has been spent in making a most judicious selection. Among essays by Sir Ben Lockspeiser, J. B. Priestley, and George Bernard Shaw (who is, incidentally, a B.I.S. member) we were pleased to find our own A. C. Clarke well to the fore with his fine paper "The Challenge of the Spaceship," which was included in the final selection and reprinted in full. We congratulate Mr. Clarke on this signal honour, so well merited. His paper, we might mention, has also been extensively quoted throughout the world and was reprinted in full in a recent issue of the *Star Weekly* (Toronto).

Earth, Moon and Planets

(By F. L. Whipple; obtainable from J. & A. Churchill, Ltd., 104, Gloucester Place, W.1. Price 18s. 6d. 293 pp. Index, photos and diagrams.)

This is one of that excellent series of works entitled the Harvard Books on Astronomy, which have been written by a team of specialists to cover the whole fundamental groundwork in astronomy. Some eight books have been published so far, other titles in the series including *The Milky Way*, *The Story of Variable Stars*, and *Atoms, Stars and Nebulæ*.

This book in the series concentrates on the major bodies of the solar system, using enough cosmogony as a cement to bind them all together and to give the reader a clear picture of the relation between our system and the rest of the universe.

Mr. Whipple has here made a very fine contribution to astronomical literature. His book embodies many novel and useful features, besides including much new material in the Mars and Lunar sections.

The context is well written and is supported by an exceptionally fine selection of astronomical photographs.

Most of the Venusian and Martian plates have never appeared for general publication before, many are very recent, and all are superlative. Both Slipher and Lowell at Flagstaff Observatory are to be congratulated on their excellent photography. Of particular note are the pictures of the "twilight arc" round Venus, and of changes in its upper atmosphere. The new photographs in the chapter on Mars are too numerous to mention.

The book begins with an introduction to the solar system and follows with a detailed analysis of the earth and moon. Then follows a discussion on Jupiter and the other solar giants, while subsequently appear the chapters on the inner planets. Here appears Slipher's rotational map of Mars, consisting of a series of 12 photographs taken after the elapse of a small angular rotational movement.

The final chapter attempts to link the planetary system with the general scheme of celestial evolution.

Five appendices follow the final chapter, of particular interest being Chapter V, which is called the "Planet Finder." This consists of tables giving the positions of the five brighter planets at all times of the year from 1940 to 1970. It works quite well in conjunction with the loose star map which is provided in a pocket on the back cover.

M. W. WHOLEY.

THE NEPTUNE ROCKET

Information has recently been published (e.g. in *Time* for July 7) on a project being undertaken for the U.S. Navy by the Glenn Martin Aircraft Company. This is the "Neptune," a high altitude research rocket of about half the weight of V.2, but with roughly twice the predicted altitude performance. Interesting features are the very high fineness ratio of "Neptune" (it is just 1 foot shorter than V.2 although only half the diameter), and the fact that control is to be obtained by tilting the whole thrust unit (chamber and nozzle) instead of by the German system of vanes in the jet. The propellants are to be alcohol and liquid oxygen, as in V.2, and first trials are anticipated late in 1948. Other relevant data are:—

Length	= 45 ft.	Weight	= 6 tons.
Diameter	= 2 ft. 8 in.	Thrust	= 11½ tons.
Time to "all burnt"	= 75 seconds.		
Velocity at "all burnt"	= 8,200 ft./sec.		
Altitude at "all burnt"	= 38 miles.		
Maximum altitude attained	= 237 miles.		

The above performance figures are with a minimum payload of 100 lb., but it is stated that a maximum payload of 1 ton is possible.

Erratum

"High Speed Flight," p. 92, December *Journal*, (1) LIFT FORCES. The second part of equation (2) should read:— $+ \frac{2\alpha}{\sqrt{1 - M^2} + 2A}$

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